An Introduction to
Accelerator Physics

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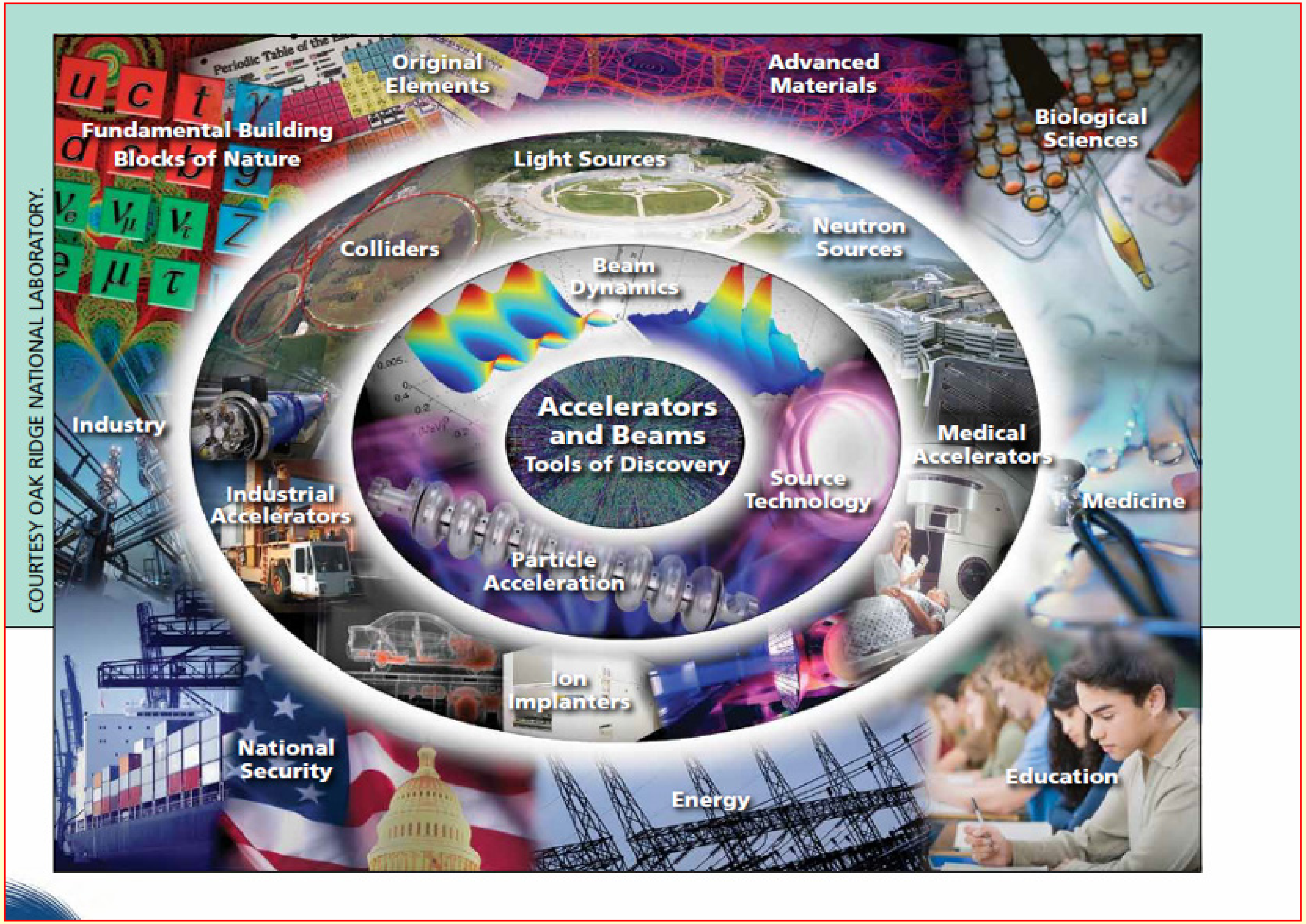
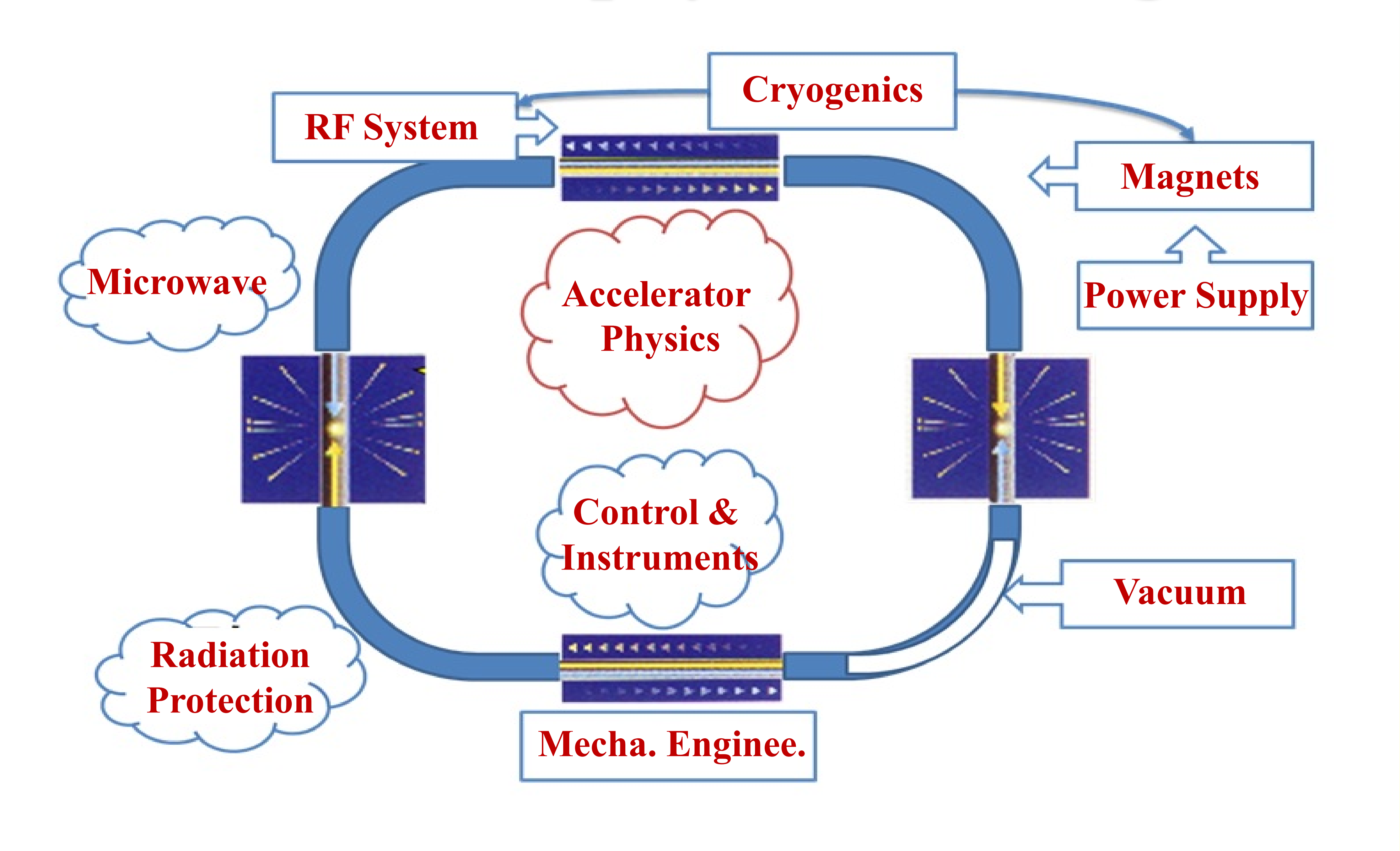
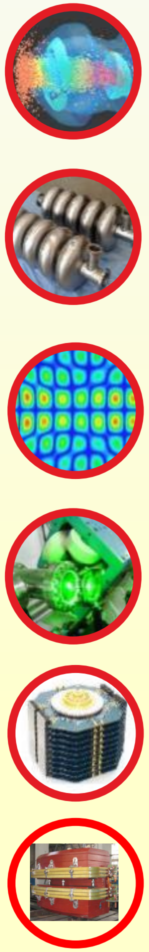
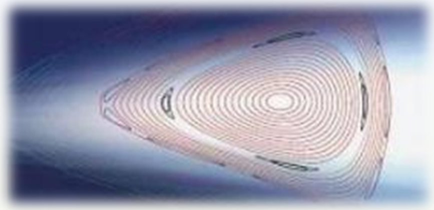
Particle Accelerators



Accelerator physics and Engineering







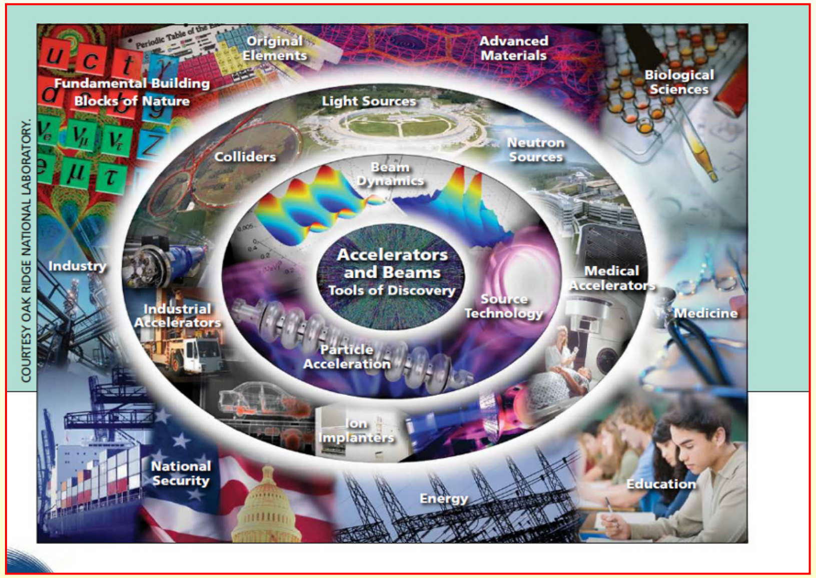
Preface

- Accelerator physics is **to study** particle beams, their motion in environments of accelerators, involving external electromagnetic fields and their interactions.
- Accelerator physics **evolves** classical mechanics, electromagnetism, statistical physics, and quantum physics.
- **Basic knowledge** of the beam physics is briefly introduced in this talk for the engineers who are working in the particle accelerator related fields, especially synchrotron radiation facilities.

An Introduction to

Accelerator Physics

- **Basic Concepts**
- **Transverse Motion**
- **Longitudinal Motion**
- **Collective Effects**
- **Synchrotron Radiation**
- **Free Electron Lasers (FEL)**



1. Basic Concepts

- Constants & Relations
- Motion in E-M Fields
- Linear accelerators
- Synchrotrons

1.1 Constants & Relations

- **Speed of light** $c = 2.99792458 \times 10^{10}$ cm/sec
- **Planck constant** $H = 6.626075 \times 10^{-34}$ J·s
- **Electron charge** $e = 1.6021773 \times 10^{-19}$ Coulumbs
- **Electron volts** $1\text{eV} = 1.6021773 \times 10^{-19}$ Joule
- **Energy and rest mass** $1\text{eV}/c^2 = 1.78 \times 10^{-36}$ kg
  - ***Electron*** $m_{0,\,e} = 0.51099906\ \text{MeV}/c^2 = 9.1093897 \times 10^{-31}$ kg
  - ***Proton*** $m_{0,\,p} = 938.2723\ \text{MeV}/c^2 = 1.6726231 \times 10^{-27}$ kg

Some Basic Relations

- **Relativistic energy** $E=mc^2=m_0\gamma c^2$
- **Relativistic momentum** $p=mv=m_0\gamma\beta c \quad \beta=v/c \quad \gamma=(1-\beta^2)^{-1/2}$
- **Energy - momentum relationship** $E^2/c^2=p^2+m_0^2c^2$
  Ultra-relativistic case: $\beta\approx 1$, $E\approx pc$
- **Kinetic energy** $T=E-m_0c^2=m_0c^2(\gamma-1)$
- **Equation of motion under Lorentz force**

$$\frac{d\vec{p}}{dt}=\vec{f}\Rightarrow m_0\frac{d}{dt}(\gamma\vec{v})=q(\vec{E}+\vec{v}\times\vec{B})$$

1.2 Motion in E-M Fields

Particle motion is governed by Lorentz force:

$$\frac{d\vec{p}}{dt} = q\left[\vec{E} + \vec{v} \times \vec{B}\right]$$

$$E^2 = \vec{p}^2 c^2 + m_0^2 c^4 \quad \Rightarrow \quad E\frac{dE}{dt} = c^2 \vec{p} \cdot \frac{d\vec{p}}{dt}$$

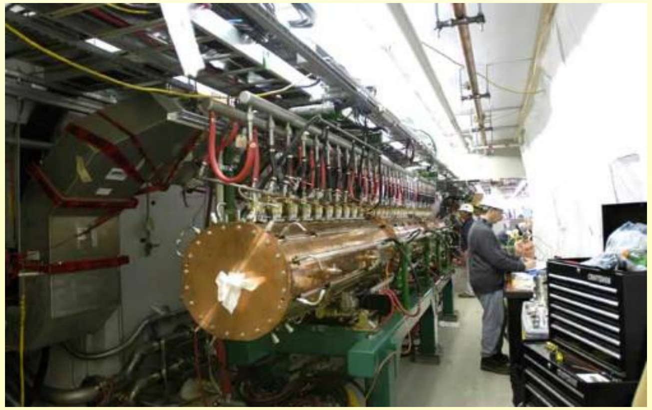
$$\Rightarrow \frac{dE}{dt} = \frac{qc^2}{E}\vec{p} \cdot \left(\vec{E} + \vec{v} \times \vec{B}\right) = \frac{qc^2}{E}\vec{p} \cdot \vec{E}$$

A magnetic field does not change a particle's energy.

Only an electric field can do this.

1.3 Linear Accelerators

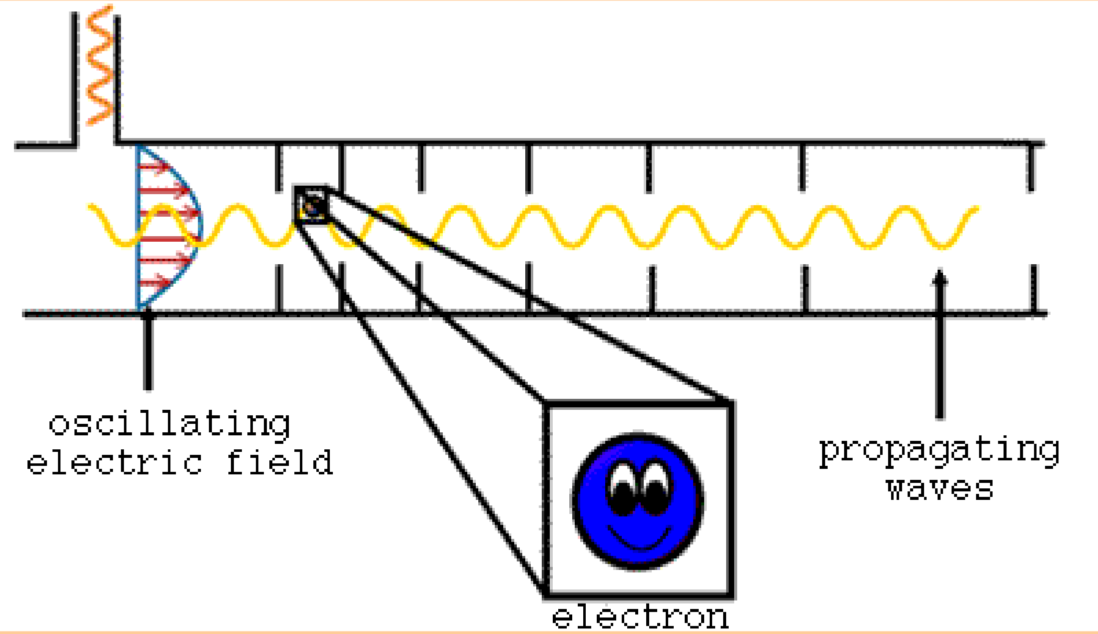
HEPS linac, IHEP



SNS Linac, Oak Ridge

- **Simplest example is a vacuum chamber with one or more DC accelerating structures with the electric field aligned in the direction of motion.**
  - ***Beam energy is limited to a few*** **MeV,** ***due to maximum electric field in material.***
- **To achieve higher energies, the electric fields are alternating at RF cavities in linacs.**
  - ***Avoids expensive bending magnets;***
  - ***No energy loss due to synchrotron radiation;***
  - ***But requires many structures for limited accelerating gradient;***
  - ***A long accelerator is needed for a high energy linac.***

Accelerating structures: Travelling wave & Standing wave

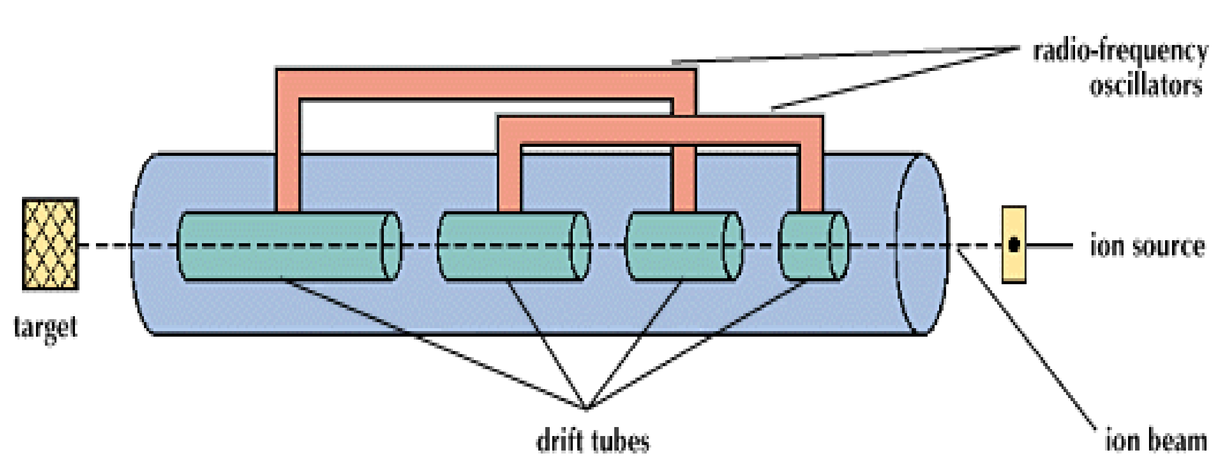


Structure 1:

- **Travelling wave structure:** particles keep in phase with the accelerating waveform.
- Phase velocity in the waveguide is greater than *c* and needs to be **reduced to the particle velocity with a series of irises inside the tube** whose polarity changes with time.
- The structure should bretty the same as **electrons at 3 MeV are already at 99% of speed of light**.
- Travelling wave structure is often applied in electron linacs .

Structure 2:

- **Standing wave structure:** A series of drift tubes alternately connected to high frequency oscillator.
- Particles **accelerated in gaps, drift inside tubes** .
- For constant frequency generator, **drift tubes increase in length as velocity increases.**
- Beam has pulsed structure.



1.4 Synchrotrons

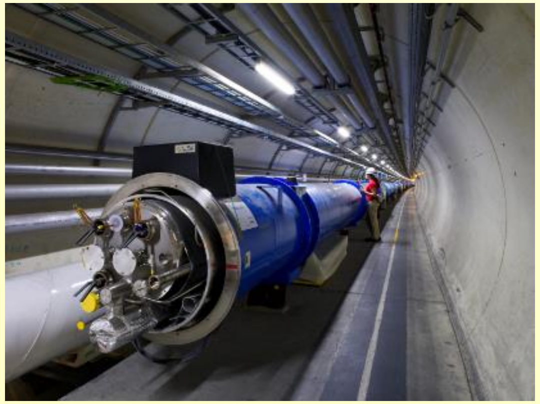
- The **principle of synchrotrons** is to vary B-field in time to match increase in energy and keep revolution radius constant and synchronism between f_{rf} and f_c.

$$B(t) = \frac{p(t)c}{Ze\rho}$$

- **Magnetic field** produced by bending magnets (*dipoles*), increases linearly with momentum.

$$f_{rf}(t) = h \cdot f_c$$

$$B\rho = \frac{p}{e} \approx \frac{E}{ce} \text{ so } E[\text{GeV}] \approx 0.3\, B[\text{T}]\, \rho[\text{m}]$$



- Large radius for high energy synchrotrons is requried for the practical limitations for magnetic fields..

e.g. LHC: $E = 7$ TeV, $B = 8.36$ T, $\rho = 2.7$ km

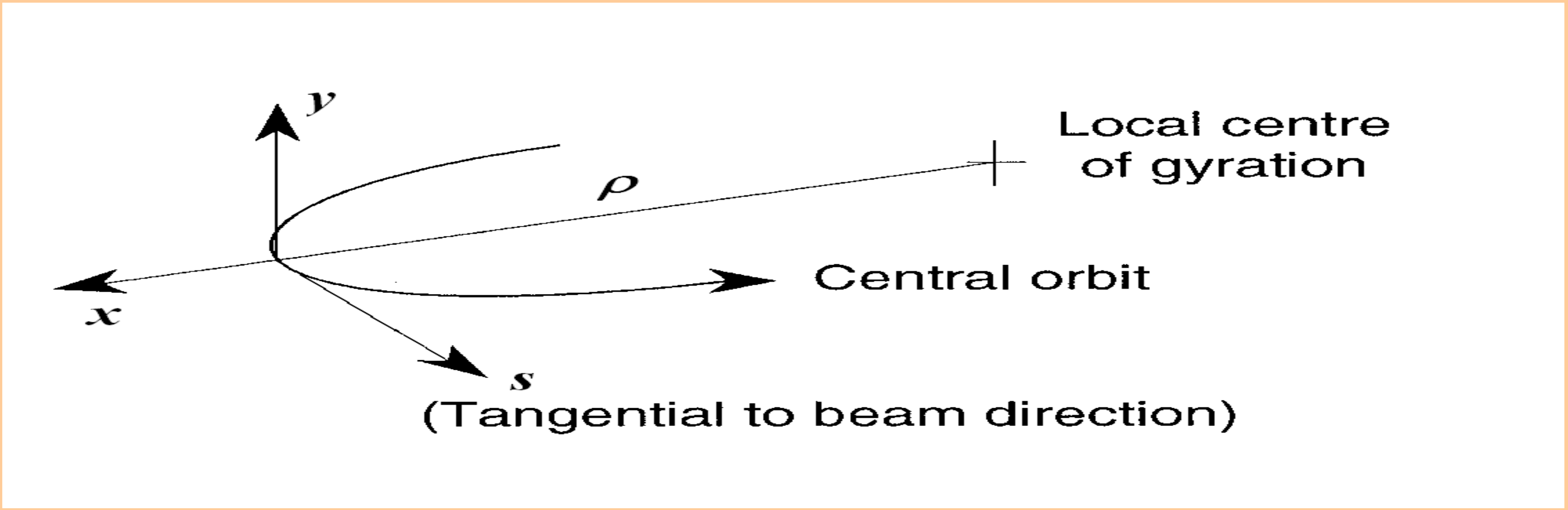
Types of Synchrotrons

- **Conventional synchrotrons:** slow or rapid cycling machine used for booster injector or fixed target experiments.
- **Storage rings:** accumulate particles and keep circulating for long periods; used for powerful machines such as synchrotron radiation facilities.
- **Colliders:** two beams circulating in opposite directions for collision to reach high center mass energy.

2. Transverse Motion

- Motion Description
- Bending
- Focusing
- Hill's Equation
- Phase Space
- Lattice
- Dispersion
- Orbit Distortion
- Coupling
- Non-linearity

2.1 Motion Description



- *s* – **longitudinal direction** — **Tangential to beam direction**
- *x* – **horizontal direction**
- *y* – **vertical direction**

Transverse directions (x and y)

2.2 Bending

- By **increasing energy *E* (hence *p*) and *B* together** in a synchrotron, it is possible to maintain a constant radius and accelerate a beam of particles.
- In a synchrotron, the confining magnetic field comes from a system of several **magnetic dipoles** forming a closed arc.
- Dipoles are **mounted apart**, separated by straight sections with vacuum chambers including equipment for focusing, acceleration, injection, insertion devices, beam monitors, and vacuum pumps, etc.

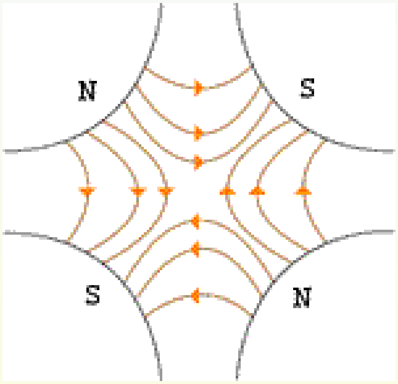
$$\rho = \left|\frac{p}{qB}\right|$$

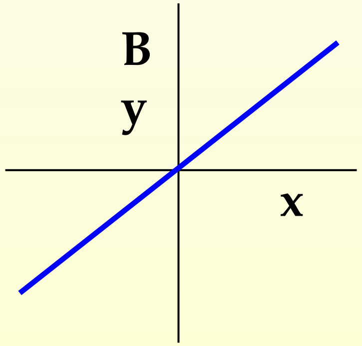
HEPS dipole in test

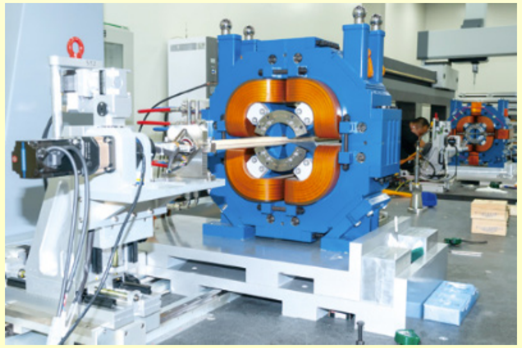
HEPS 7BA cell

2.3 Focusing

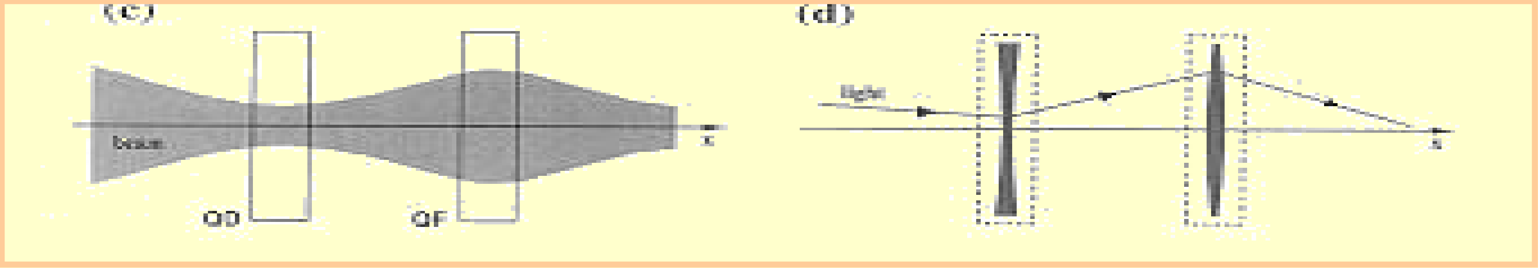
- A sequence of focusing-defocusing fields provides a **strong focusing force**.
- Quadrupoles focus horizontally, defocus vertically or vice versa. **Forces are linearly** proportional to displacement from axis.
- A succession of opposed elements enable particles to follow **stable trajectories**, making small (betatron) oscillations about the design orbit.







HEPS quadrupole



2.4 Hill's Equation

- **Hill's Equation:**

$$x'' + k_x(s)x = 0, \quad y'' + k_y(s)y = 0$$

- **Equations of motion in accelerator components :**
  - ***Drift section:*** $x'' = 0, \quad y'' = 0$ $\quad M(s/s_0) = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$
  - ***Dipole:*** $x'' + \dfrac{1}{\rho^2}x = 0, \quad y'' = 0$
  - ***Quadrupole:*** $x'' + k\,x = 0, \quad y'' - k\,y = 0$
  - ***Sextupole :*** $x'' + ks(x^2 - y^2) = 0, \quad y'' - 2ksxy = 0$

Solution of the Hill's Equation

- **Hill's equation**

It is with linear-periodic coefficients

$$\frac{d^2u}{ds^2} + k(s)u = 0$$

where $k(s) = -\frac{1}{B\rho}\frac{dB_y}{dx}$

and *u* denotes *x* and *y*

- **Like restoring constant in simple harmonic motion, the solution is**

$$u(s) = \sqrt{\varepsilon\beta(s)}\sin(\phi(s) + \phi_0)$$

where $\phi(s) = \int \frac{ds}{\beta(s)}$ **is betatron phase**

- $\beta(s)$ **– envelope function**

  is behavior of the machine

- ε **– Emittance**

  is behavior of the beam

- **Physical meaning (H and V planes)**
  - **Beam envelope**

    $$u_{\text{env}}(s) = \sqrt{\varepsilon\beta(s)}$$

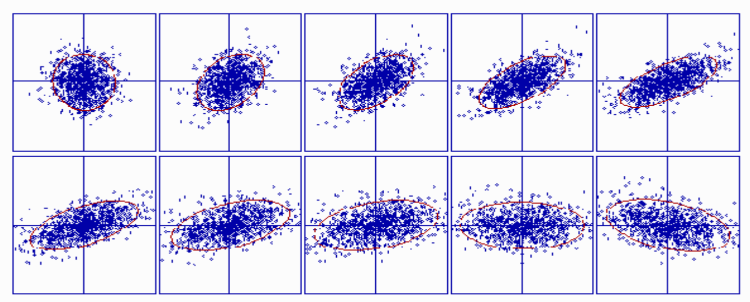
  - **Maximum excursions**

    $$u'(s) = \sqrt{\varepsilon / \beta(s)}$$

2.5 Phase Space

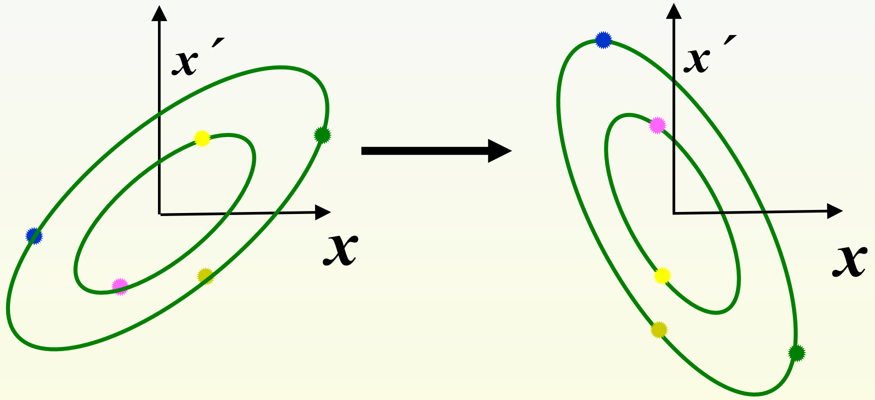
- **Under linear forces, any particle moves on an ellipse in phase space (x, x').**
- **Ellipse rotates in accelerator lattice, but its area is preserved:**

Emittance



- ***Acceptance: $A_{x,y} > \varepsilon_{x,y}$***

$$A_x = \left(\frac{X^2}{\beta_x}\right)_{\max}, \quad A_y = \left(\frac{Y^2}{\beta_y}\right)_{\max}$$



- **General equation of ellipse is**

$$\beta x'^2 + 2\alpha x x' + \gamma x^2 = \varepsilon$$

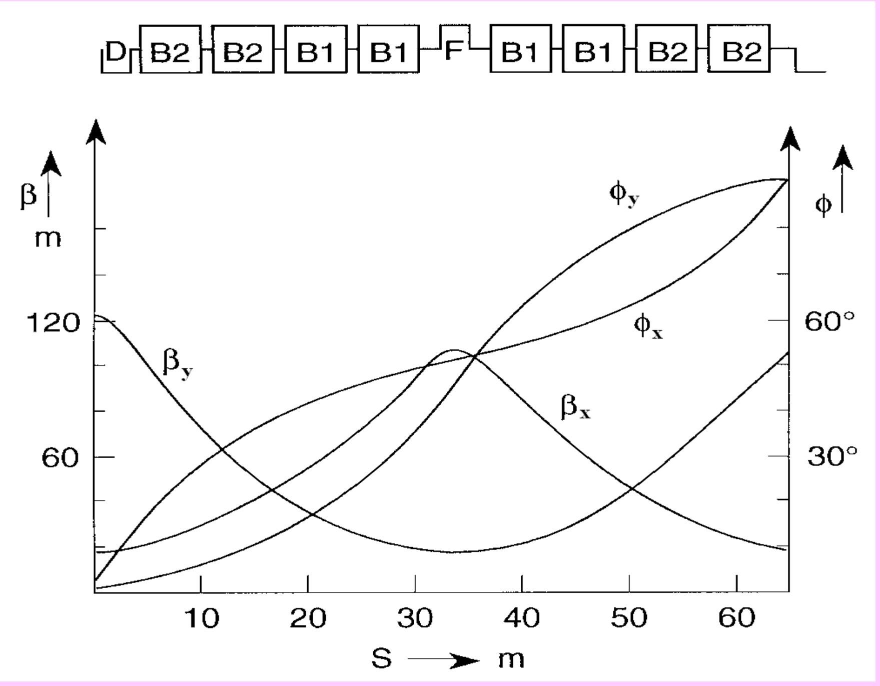
- **α, β, γ are functions of distance s (Twiss parameters), and ε is a constant. Area of ellipse = $\pi\varepsilon$.**
- **For non-linear beams one can use 95% emittance ellipse or rms emittance**

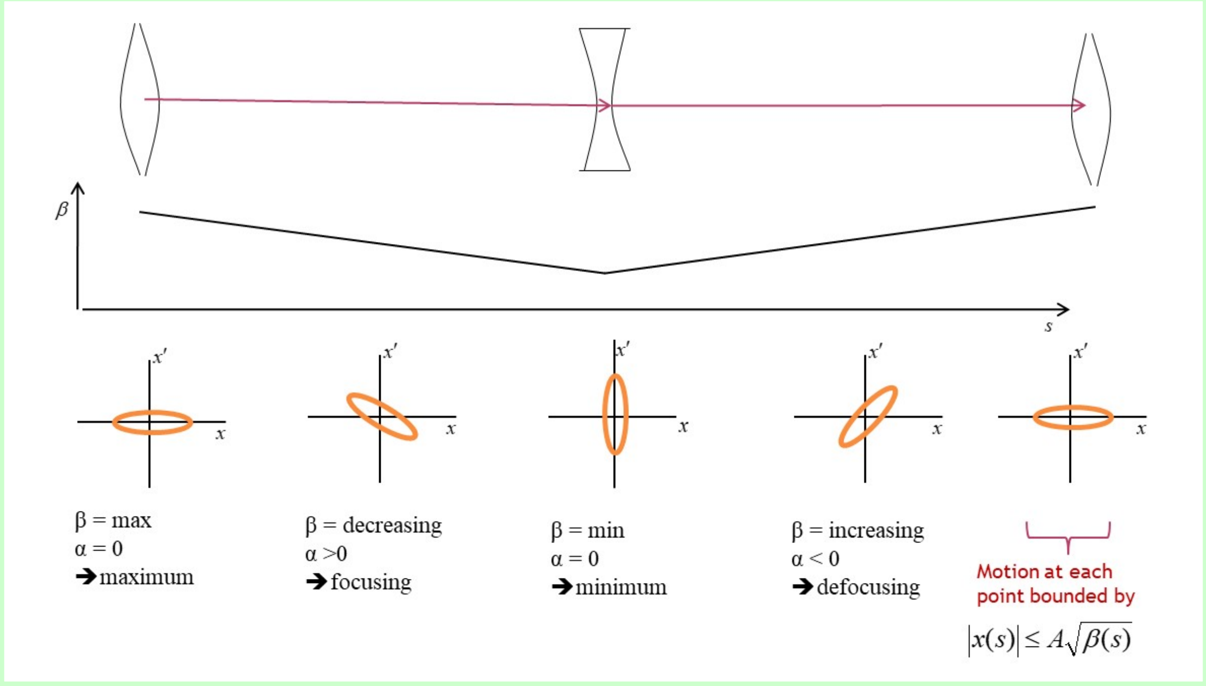
$$\varepsilon_{rms} = \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$$

(statistical definition)

2.6 Lattice

- Lattice is defined as the **pattern** of focusing magnets, bending magnets and straight sections connecting in between;
- It has a **strong influence on aperture of vacuum chambers** and thus other systems of the accelerator.





Matrix formalism

- As a consequence of the **linearity of Hill's equations**, we can describe the evolution of the trajectories in a lattice by means of **linear transformations**

$$\begin{pmatrix} y(s) \\ y'(s) \end{pmatrix} = \begin{pmatrix} C(s) & C'(s) \\ S(s) & S'(s) \end{pmatrix} \begin{pmatrix} y(s_0) \\ y'(s_0) \end{pmatrix} = M(s/s_0) \begin{pmatrix} y(s_0) \\ y'(s_0) \end{pmatrix}$$

- In terms of the amplitude and phase function the **transfer matrix** is

$$M(s_2/s_1) = \begin{pmatrix} \sqrt{\dfrac{\beta(s)}{\beta_0}}(\cos\Delta\phi + \alpha_0 \sin\Delta\phi) & \sqrt{\beta(s)\beta_0}\sin\Delta\phi \\ -\dfrac{(\alpha(s)-\alpha_0)\cos\Delta\phi + (1+\alpha(s)\alpha_0)\sin\Delta\phi}{\sqrt{\beta(s)\beta_0}} & \sqrt{\dfrac{\beta_0}{\beta(s)}}[\cos\Delta\phi - \alpha(s)\sin\Delta\phi] \end{pmatrix}$$

where $\alpha = \mathrm{d}\beta/\mathrm{ds}$， $\gamma = (1+\alpha^2)/\beta$.

- For a periodic machine the **transfer matrix over a lattice period** reduces to

$$M(s_0 + L/s_0) = \begin{pmatrix} \cos\mu + \alpha_0 \sin\mu & \beta_0 \sin\mu \\ -\gamma \sin\mu & \cos\mu - \alpha_0 \sin\mu \end{pmatrix}$$

Matrix formalism

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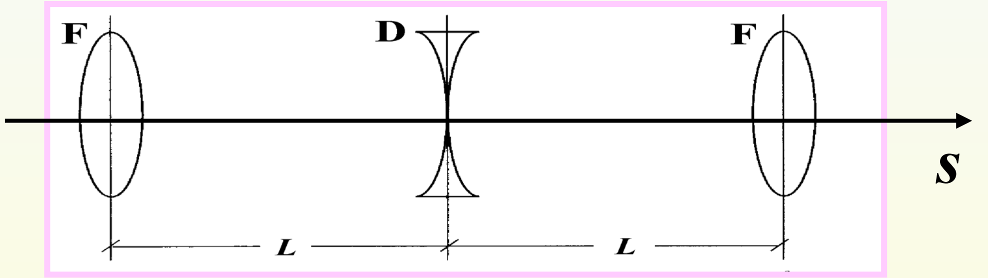
Example: FODO Lattice

- The matrix for one period between mid-planes of F magnet in thin lens approximation is



$$M_{x,y} = \begin{pmatrix} 1 & 0 \\ \mp 1/2f & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \pm 1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \mp 1/2f & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 - L^2/2f^2 & 2L(1 \pm L/2f) \\ -L/2f^2(1 \mp L/2f) & 1 - L^2/2f^2 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ \gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

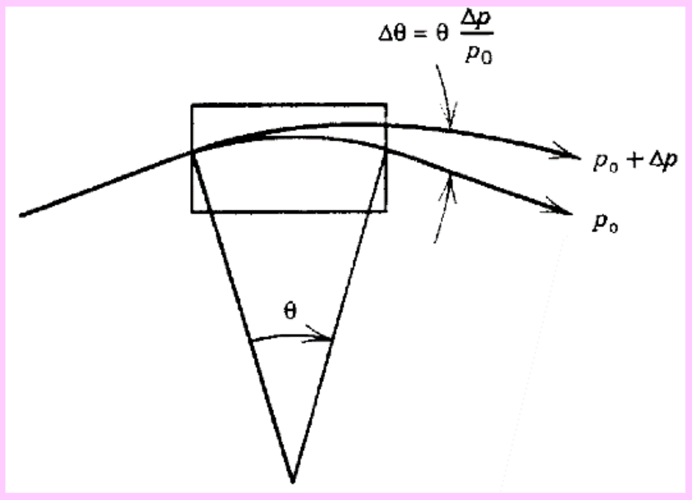
$$\mu_{x,y} = \cos^{-1}\left(1 - \frac{L^2}{2f^2}\right)$$

$$\beta_{x,y} = 2L\frac{1 \pm \sin(\mu/2)}{\sin\mu} = \frac{1}{\gamma_{x,y}}$$

$$\alpha_{x,y} = 0 \qquad Q_{x,y} = (N_c \cdot \mu_{x,y}/2\pi)$$

2.7 Dispersion

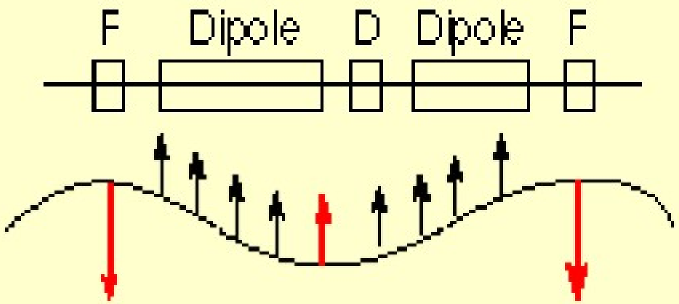
- The dispersion has its origin that **a particle of higher momentum is deflected through a less angle** in a bending magnet.



- It was shown that the equations of motion of a charged particle is a **linear Hill's equation**

$$x'' + \left(\frac{1}{\rho^2} - k(s)\right)x = \frac{1}{\rho}\frac{\Delta p}{p_0}$$

- The **solutions of the equation** can be written in terms of the optics functions



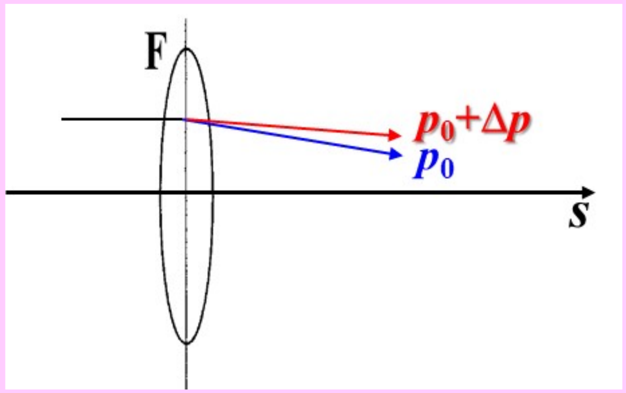
$$x(s) = \sqrt{\varepsilon\beta(s)}\cos(\phi(s) - \phi_0) + \frac{\Delta p}{p_0}D(s)$$

$$D = \frac{\sqrt{\beta(s)}}{2\sin\pi\nu}\oint\frac{\sqrt{\beta(\bar{s})}}{\rho(\bar{s})}\cos[\pi\nu - \phi(\bar{s}) - \phi(s)]d\bar{s}$$

Chromaticity

- Chromaticity is defined as betatron tune changes with momentum spread:

$$\xi_{x,y} = \frac{\Delta Q_{x,y}}{\Delta p/p_0}$$



- The chromaticity has its origin that **a particle of higher momentum is focused less** in a quadrupole magnet.

- The Hill's equation is written as:

$$u'' + (k + \Delta k)u = 0$$ where u denotes x or y, and $\Delta k = -k \cdot \Delta p/p_0$

- The chromaticity of the synchrotron is derived as: $\xi_{x,y} = -\frac{1}{4\pi}\oint \beta_{x,y}(s)k_{x,y}(s)ds$

- Non-zero chromaticity will make tune spread for momentum spread, and negative chromaticity will cause **head-tail instability** when $\gamma > \gamma_t$.

- Sextupoles are used for **correct chromaticity**, while the nonlinear effects need to be mitigated by detail simulation to obtain **desired dynamic aperture**.

2.8 Orbit Distortion

- **Dipole errors** may cause orbit distortion of particle beams.

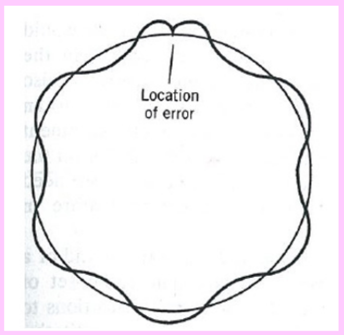
Element	Source	Field Error	Deflection	Direction
Dipole (angle=θ)	Field error	$\langle \Delta B/B \rangle$	$\theta \langle \Delta B/B \rangle$	x
	Tilt	$\langle \Delta\theta_z \rangle$	$\theta \langle \Delta\theta_z \rangle$	y
Quadrupole (Kl)	Displacement	$\langle \Delta x, \Delta y \rangle$	$Kl \cdot \langle \Delta x, \Delta y \rangle$	x, y

- Similar to dispersion case, the **equation of motion** is written as

$$y'' + ky = \frac{1}{B\rho}\frac{\Delta B(s)}{B}$$

- The **solutions** are $y(s) = \sqrt{\varepsilon\beta(s)}\cos(\phi(s) - \phi_0) + y_{cod}(s)$

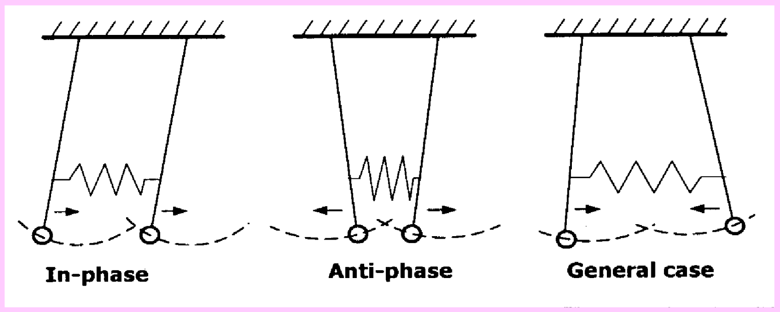
$$y_{COD} = \frac{\sqrt{\beta(s)}}{2\sin\pi\nu}\oint\frac{\Delta B(\bar{s})\sqrt{\beta(\bar{s})}}{B\rho(\bar{s})}\cos[\pi\nu - \phi(\bar{s}) - \phi(s)]d\bar{s}$$



Measured by BPM's
Corrected by dipoles

2.9 Linear Coupling

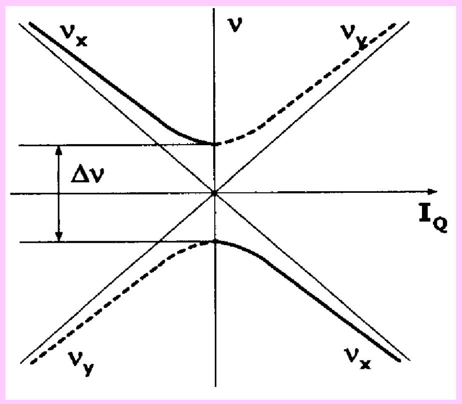
- Coupling is the phenomena that **energy exchange between two oscillators**.
- In accelerators, the **horizontal and vertical motions** are also coupled:



$$x'' + k_x x = -(k + M'/2)y - My'$$
$$y'' + k_y y = -(k - M'/2)x - Mx'$$

Where

$$k = -\frac{1}{2B\rho}\left(\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y}\right)_0, \qquad M = -\frac{B_s}{B\rho}$$



- x-y coupling can be **compensated** with skew quadrupoles and anti-solenoids.
- It can be **measured** with tune approaching.

$$\kappa = \frac{\varepsilon_y}{\varepsilon_x} = \frac{(\Delta\nu)^2}{(\nu_x - \nu_y)^2 + 2(\Delta\nu)^2}$$

2.10 Non-linearity

- There are **higher order terms** in the expansion of magnetic field of magnets:

$$B_y + iB_x = B_0\rho_0\left[\sum_{n=1}^{M}\frac{k_n(s)+ij_n(s)}{n!}(x+iy)^n\right]$$

$$k_n = \frac{1}{B_0\rho_0}\frac{\partial^n B_y}{\partial x^n}\bigg|_{(0,0)}$$ Normal multipoles

$$j_n = \frac{1}{B_0\rho_0}\frac{\partial^n B_x}{\partial x^n}\bigg|_{(0,0)}$$ Skew multipoles

- The Hill's equations acquire additional **nonlinear terms**:

$$\frac{d^2x}{ds^2}+\left(\frac{1}{\rho^2(s)}-k_1(s)\right)x = \mathrm{Re}\left[\sum_{n=2}^{M}\frac{k_n(s)+ij_n(s)}{n!}(x+iz)^n\right]$$

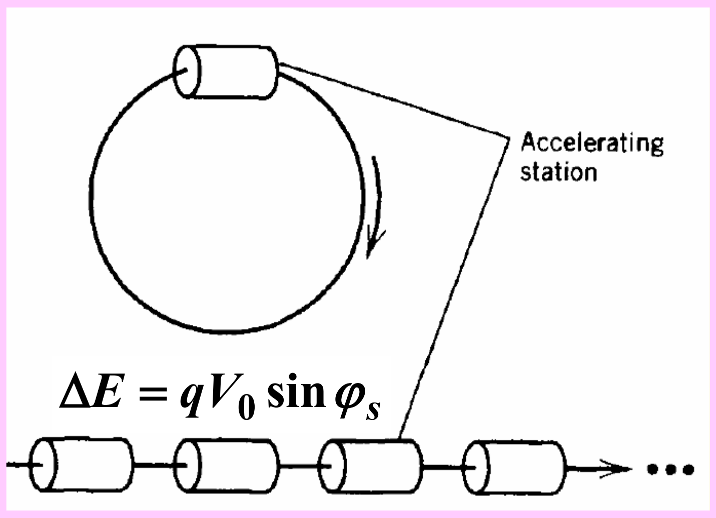
$$\frac{d^2z}{ds^2}+k_1(s)z = -\mathrm{Im}\left[\sum_{n=2}^{M}\frac{k_n(s)+ij_n(s)}{n!}(x+iz)^n\right]$$

- There is no analytical solution available in general, and the **equations have to be solved by tracking or perturbative analysis.**

3. Longitudinal Motion

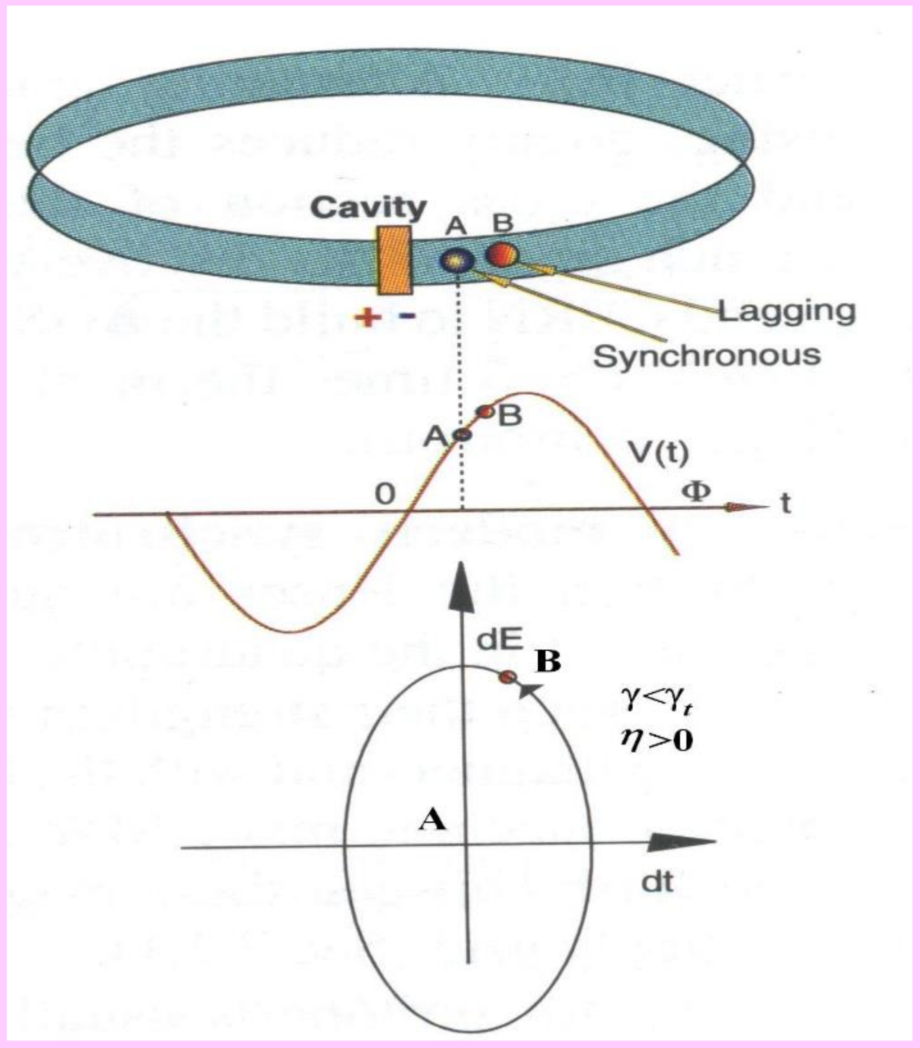
- RF Cavities & Acceleration
- Synchrotron Oscillation
- Bunches and Buckets
- γ-Transition

3.1 RF Cavities & Acceleration



- RF cavities are necessary condition for beam **acceleration**.
- Both linear and circular accelerators use electromagnetic fields oscillating in **resonant cavities** to apply the accelerating force.
- **In linacs** – particles follow straight path through series of cavities.
- **In circular accelerators** – particles follow circular path in *B*-field and particles return to same accelerating cavity each time around.

3.2 Synchrotron Oscillation



- **This is a rigid pendulum**

$$\ddot{\phi} = -\frac{2\pi V_0 h \eta f_0^2}{E_0 \beta^2 \gamma}(\sin\phi - \sin\phi_s)$$

- **For small amplitudes**

$$\ddot{\phi} + \frac{2\pi V_0 h \eta f_0^2}{E_0 \beta^2 \gamma}\Delta\phi = \ddot{\phi} + \Omega_s^2 \Delta\phi = 0$$

- **Synchrotron frequency**

$$f_s = \sqrt{\frac{\eta h V_0 \cos\phi_s}{2\pi E_0 \beta^2 \gamma}}\, f_0$$

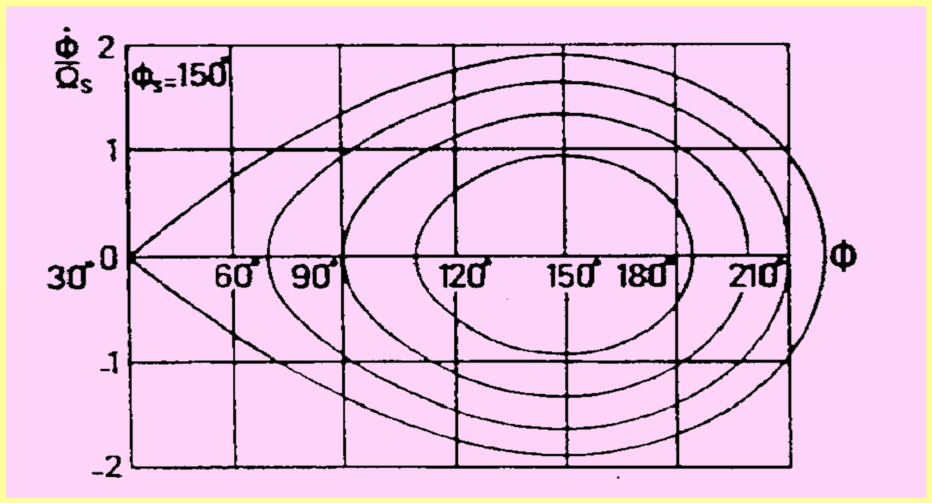
- **Synchrotron tune**

$$\nu_s = \frac{f_s}{f} = \sqrt{\frac{\eta h V_0 \cos\phi_s}{2\pi E_0 \beta^2 \gamma}}$$

3.3 Bunches and Buckets

- For large amplitudes

$$\ddot{\phi} = -\frac{\Omega_s^2}{\cos\phi_s}(\sin\phi - \sin\phi_s)$$



- Integrated becomes an invariant

$$\frac{\dot{\phi}^2}{2} - \frac{\Omega_s^2}{\cos\phi_s}(\cos\phi + \phi\sin\phi_s) = const.$$

- The equation of each separatrix is

$$\frac{\dot{\phi}^2}{2} - \frac{\Omega_s^2}{\cos\phi_s}(\cos\phi + \phi\sin\phi_s) = -\frac{\Omega_s^2}{\cos\phi_s}\left[\cos(\pi - \phi_s) + (\pi - \phi_s)\sin\phi_s\right].$$

- And the half height is

$$(\Delta E/E_s)_{\max} = \pm\beta\left\{\frac{eV_0}{\pi h\eta E_s}G(\phi_s)\right\}^{1/2} \qquad G(\phi_s) = \left[2\cos\phi_s - (\pi - 2\phi_s)\sin\phi_s\right]$$

- Bucket size should be larger than bunch size [$\Delta E/E_s$ and $\Delta\phi$ ($\Delta\tau$, Δz)] ⇨ high V_0

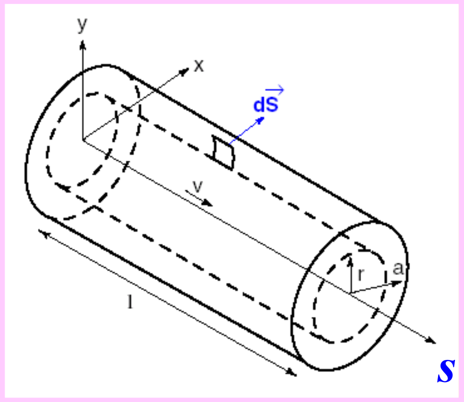
4. Collective Effects

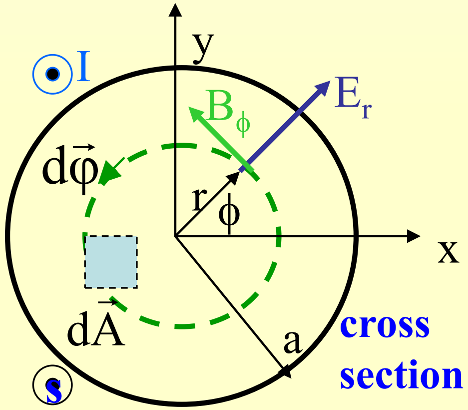
- **Space Charge**
- **Wake Fields and Impedance**
- **Beam Instabilities**
- **Beam-beam Effects**

4.1 Space Charge

A charged particle in a beam undergoes electromagnetic field from other particles

η…charge density in C/m^3
λ… constant line charge $\pi a^2 \eta$
I…constant current $\lambda\beta c = \pi a^2 \eta\beta c$
a…beam radius





Electric

$$\text{curl}\,\vec{B} = \mu_0 \vec{J} \qquad \text{div}\,\vec{E} = \frac{\eta}{\varepsilon_0}$$

$$\iiint \text{div}\,\vec{E}\, dV = \iint \vec{E}\, d\vec{S}$$

Volume element

Magnetic

$\vec{B} = B_\varphi$ $\quad \vec{E} = E_r$

Current density ($\beta c \eta$)

$$\oint \vec{B}\, r\, d\vec{\varphi} = \iint curl\,\vec{B}\, d\vec{A}$$

Apply these integrals over

cylinder radius r length l

$$r^2 \pi l \frac{\eta}{\varepsilon_0} = E_r 2 r \pi\pi$$

cross section radius r

$$B_\varphi 2 r\pi = \mu_0 r^2 \pi \beta c \eta$$

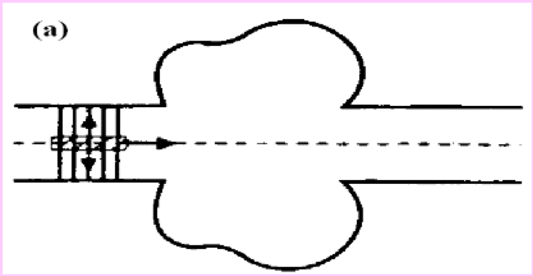
$$E_r = \frac{I}{2\pi\varepsilon_0 \beta c}\frac{r}{a^2}$$

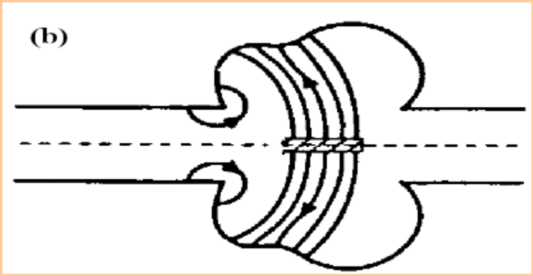
$$B_\varphi = \frac{I}{2\pi\varepsilon_0 c^2}\frac{r}{a^2}$$

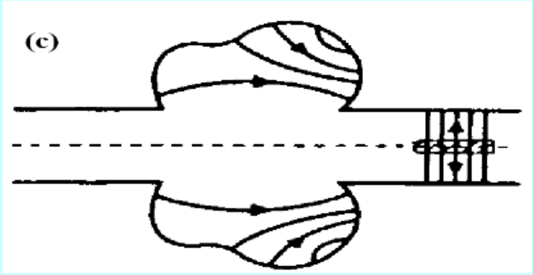
For $\beta \to 1$ ($\gamma >> 1$) $\quad F_E = qE_r = -F_B = -q \cdot B_\phi \cdot v$ so $\Sigma F = 0$

4.2 Wake Fields and Impedance

- **Wake field** will be **driven by a beam** when there is a discontinuity in the vacuum chamber:







$$\vec{F}_{\perp}(r,\theta,z) = -eI_m W_m(z) m r^{m-1}(\hat{r}\cos m\theta - \hat{\theta}\sin m\theta)$$

$$F_{\parallel}(r,\theta,z) = -eI_m W'_m(z) r^m \cos m\theta$$

- **Impedances** are just Fourier transforms of wake functions:

$$Z_m^{\perp}(\omega) = \frac{i}{v/c}\int \frac{dz}{v} e^{-i\omega z/v} W_m(z) \quad , \quad Z_m^{\parallel}(\omega) = \int_{-\infty}^{\infty} \frac{dz}{v} e^{-i\omega z/v} W'_m(z)$$

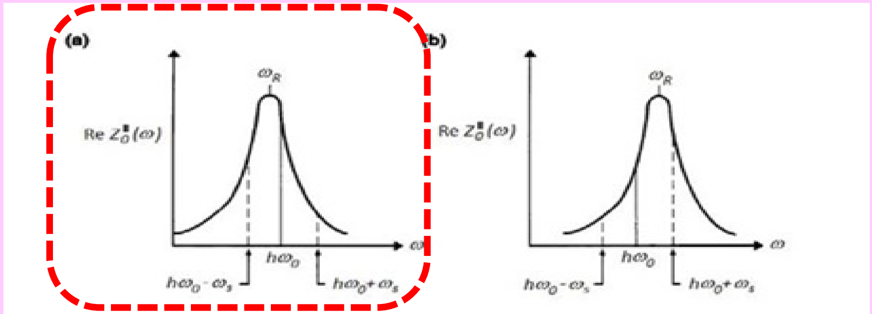
Impedance $Z = Z_r + iZ_i$

Induced voltage $V \sim I_w Z = -I_B Z$

V acts back on the beam
⇨ **Instabilities**

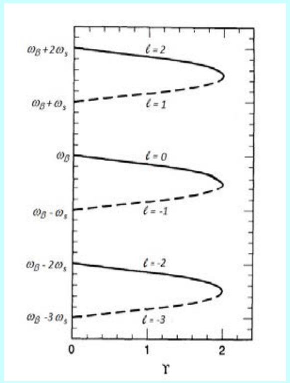
4.3 Instabilities

- **Robinson instability**
  - Caused by fundamental cavity mode
  - **Cure:** Beam will be stabilized by properly tuning the cavity ($f_{rf} < hf_0$ for $\gamma > \gamma_t$).



Resonance frequency above and below transition

- **Head-tail Instability**
  - Caused by transverse wake field
  - **Cure:** Beam will be stabilized by properly setting chromaticity ($\xi > 0$ for $\gamma > \gamma_t$).
- **Strong head-tail Instability**
  - Caused by transverse wake as counterpart of beam break-up in a linac.
  - **Cure:** control of broadband transverse impedance.



- **Potential-well distortion and microwave instability**
  - **Cure:** control of broadband longitudinal impedance.
- **Ion trapping and fast ion instability** – better vacuum and proper bunch pattern
- **Coupled bunch instability**
  - Multi-bunch effects caused by long-range transverse & longitudinal wakes
  - **Cure:** control of narrowband impedance and applying feedback systems.

5. Synchrotron Radiation

- Principle of synchrotron radiation
- Radiation damping
- Radiation fluctuation
- Synchrotron radiation sources

5.1 Principle of Synchrotron radiation

- The **electromagnetic radiation** emitted when the charged particles are accelerated (deflected).

- The particle **loses the energy of** U_0 in one turn:

$$U_0 = \frac{e^2\gamma^4}{3\varepsilon_0\rho}$$

- For electrons:

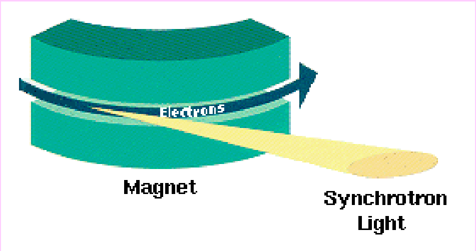
$$U_0(keV) = 88.46\frac{E(GeV)^4}{\rho(m)}$$

e.g. LEP2: E=100GeV, $\rho = 3100$ m, U_0=1.85 GeV

- For protons:

$$U_0(keV) = 7.78\times10^{-12}\frac{E(GeV)^4}{\rho(m)}$$

e.g. LHC: E=7000GeV, ρ= 2804 m, U_0= 6.66 keV

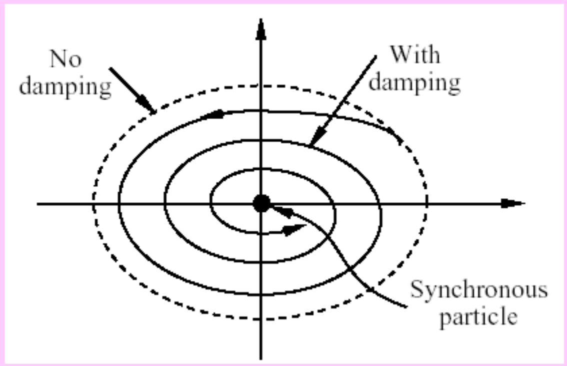


5.2 Radiation damping

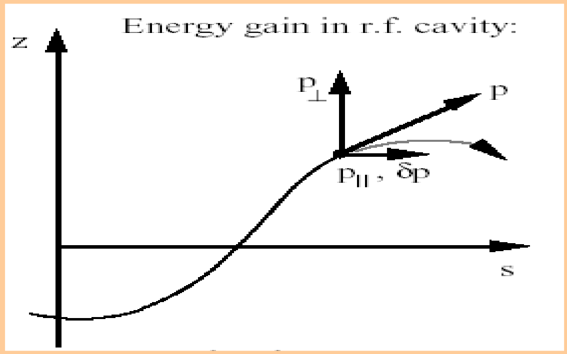
- In storage rings, synchrotron radiation loss is compensated by the RF fields:

$$U_0 = eV_0 \sin(\varphi_s)$$

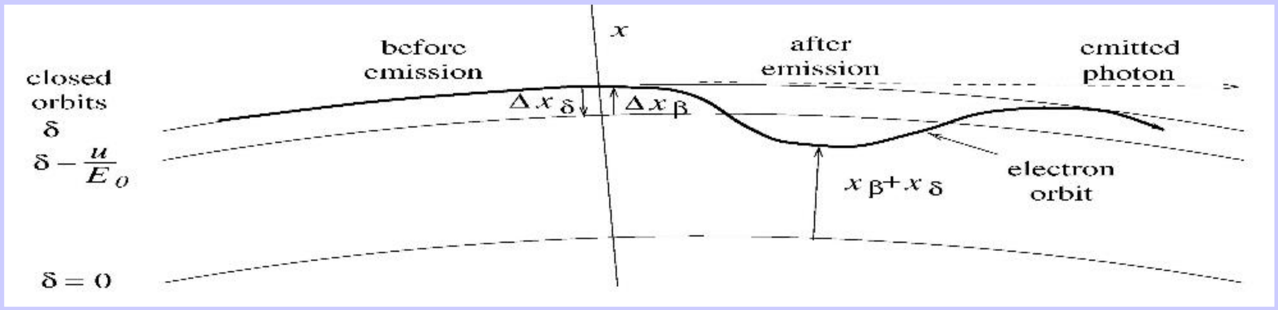
- This will cause radiation damping: $\frac{d^2\varepsilon}{dt^2} + \frac{2}{\tau_s}\frac{d\varepsilon}{dt} + \omega_s^2\varepsilon = 0$

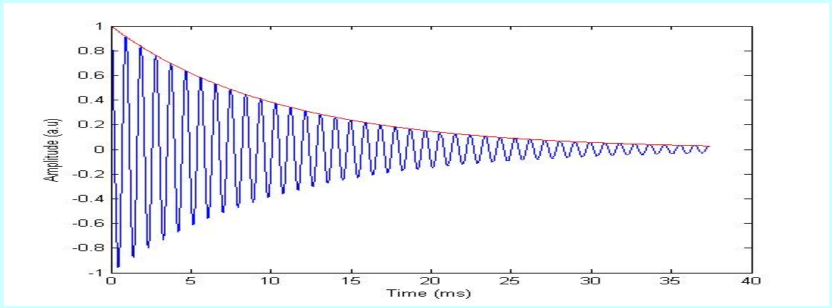


- **Longitudinal:** higher the energy of particle more the SR loses;
- **Vertical:** radiated momentum is with $P_\perp$, while RF compensates $P_{\|}$;



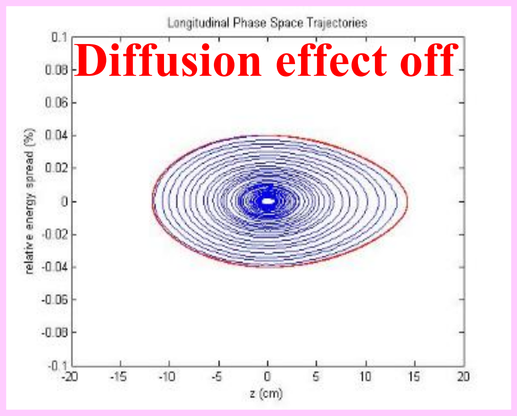
- **Horizontal:** After the photon emission $\delta x_\beta + \delta x_\varepsilon = 0$

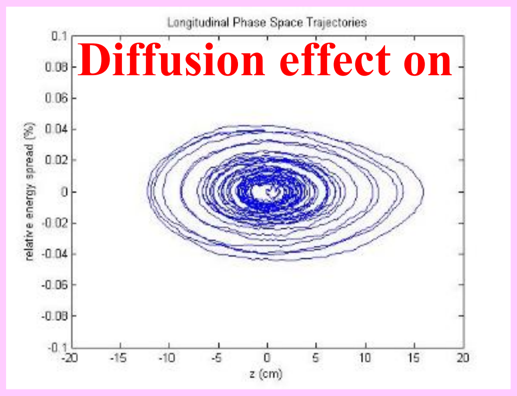




5.3 Radiation fluctuation

- If it is with **pure damping**, the emittance would be getting to zero in x, y and s directions.
- However, the radiated energy is **emitted in quanta**: each quantum carries an energy $u = \hbar\omega$;
- The emission process is instantaneous and the time of emission of individual quanta is **statistically independent**;
- Radiation damping combined with radiation fluctuation determine the **equilibrium beam distribution** and therefore finite emittance, beam size, energy spread and bunch length.





$$\varepsilon_x = \frac{55}{32\sqrt{3}}\frac{\hbar}{mc}\frac{\gamma^2}{J_x}\frac{\langle H/\rho^3\rangle}{\langle 1/\rho^2\rangle}$$

$$\sigma_E = \left(\frac{55}{64\sqrt{3}}\frac{\hbar}{\rho mc}\right)^{1/2}\cdot\gamma$$

$$\sigma_z = \frac{\alpha c}{\Omega_s}\sigma_E$$

J_x is damping partition numbers, H is dispersion invariant: $H(s) = \gamma D_x^2 + 2\alpha D_x D_x' + \beta D_x'^2$

https://lightsources.org/

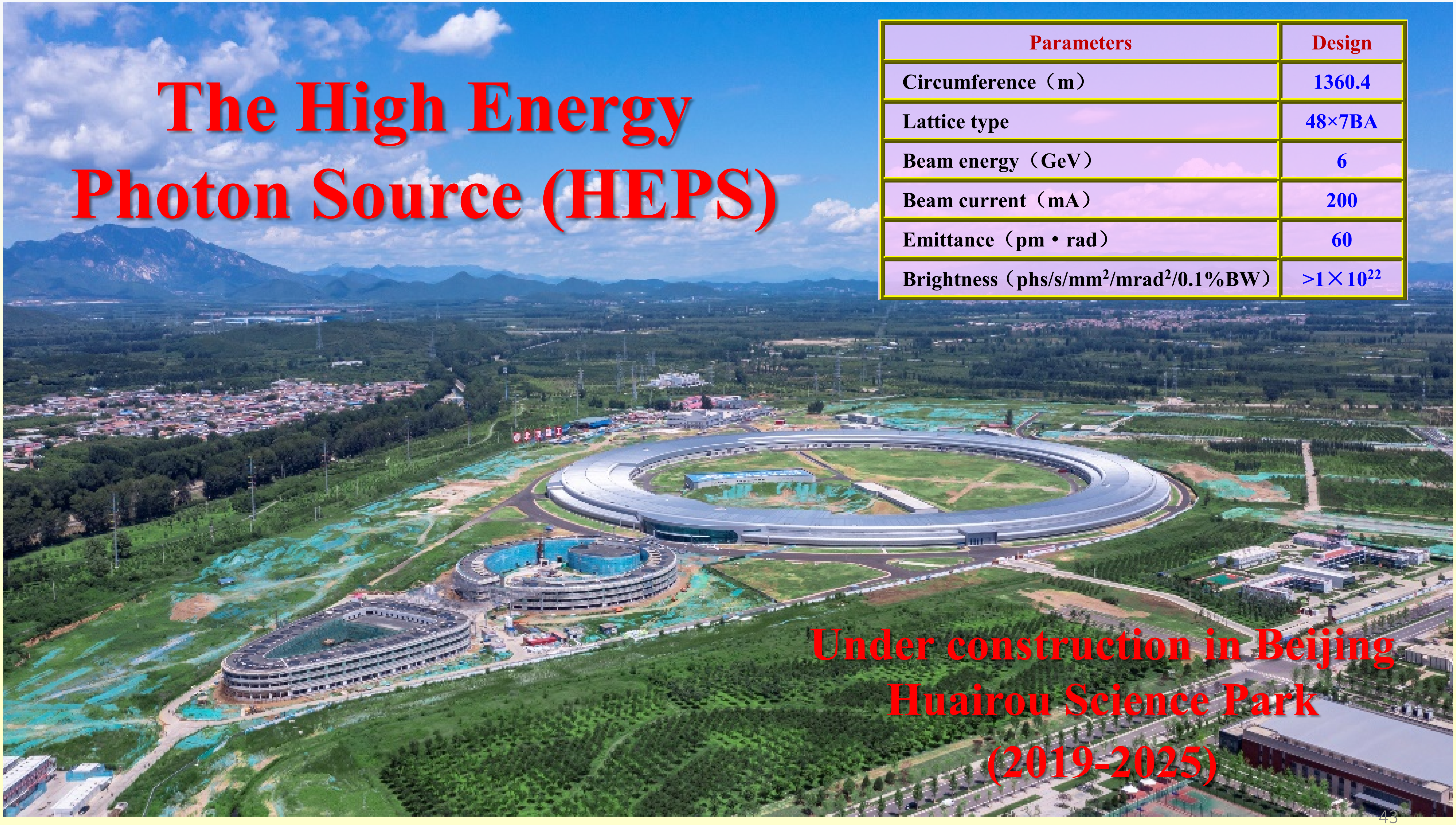
5.4 Synchrotron radiation facilities

There are more than 50 synchrotron radiation facilities in the world

(operational or under construction)

The High Energy Photon Source (HEPS)

Parameters	Design
Circumference（m）	1360.4
Lattice type	48×7BA
Beam energy（GeV）	6
Beam current（mA）	200
Emittance（pm・rad）	60
Brightness（$phs/s/mm^2/mrad^2/0.1\%BW$）	$>1\times10^{22}$

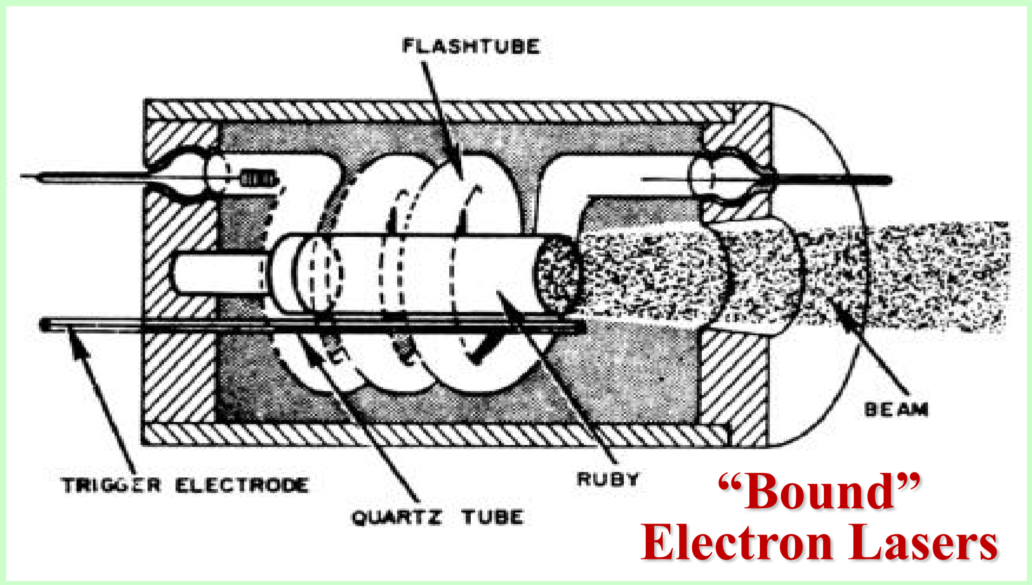


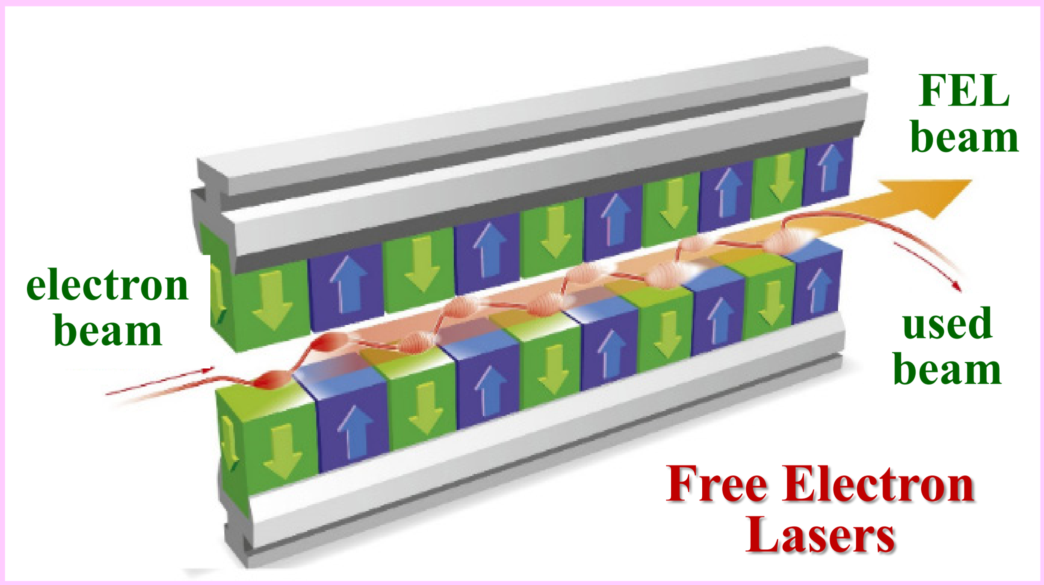
Under construction in Beijing Huairou Science Park (2019-2025)

5. Free Electron Laser (FEL)

- From traditional laser to FEL
- From SR to FEL
- FEL’s in the world

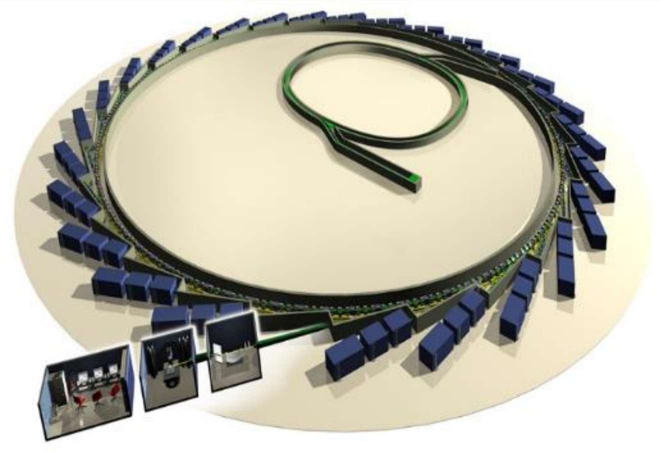
5.1 Traditional laser and FEL

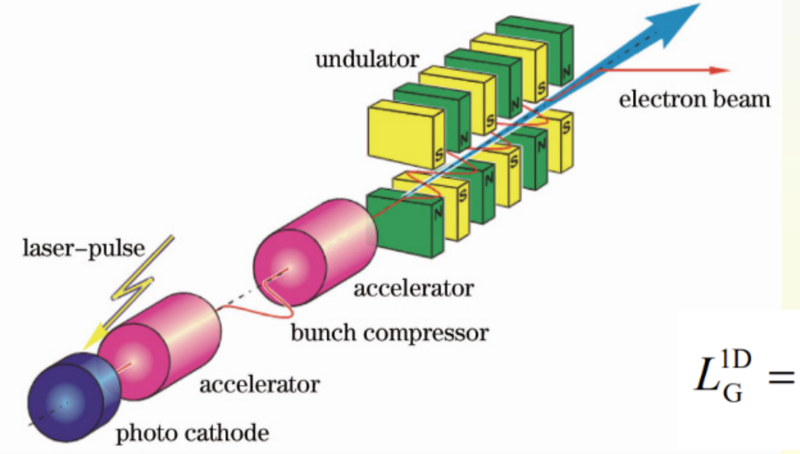




- **Traditional laser** gains energy from energy level transition in the "bound" electrons in atoms.
- **FEL** converts the energy of free electrons of accelerators into laser power.
- FEL's may provide high power lasers from THz to X-rays.

6.2 From SR to FEL





Resonant condition

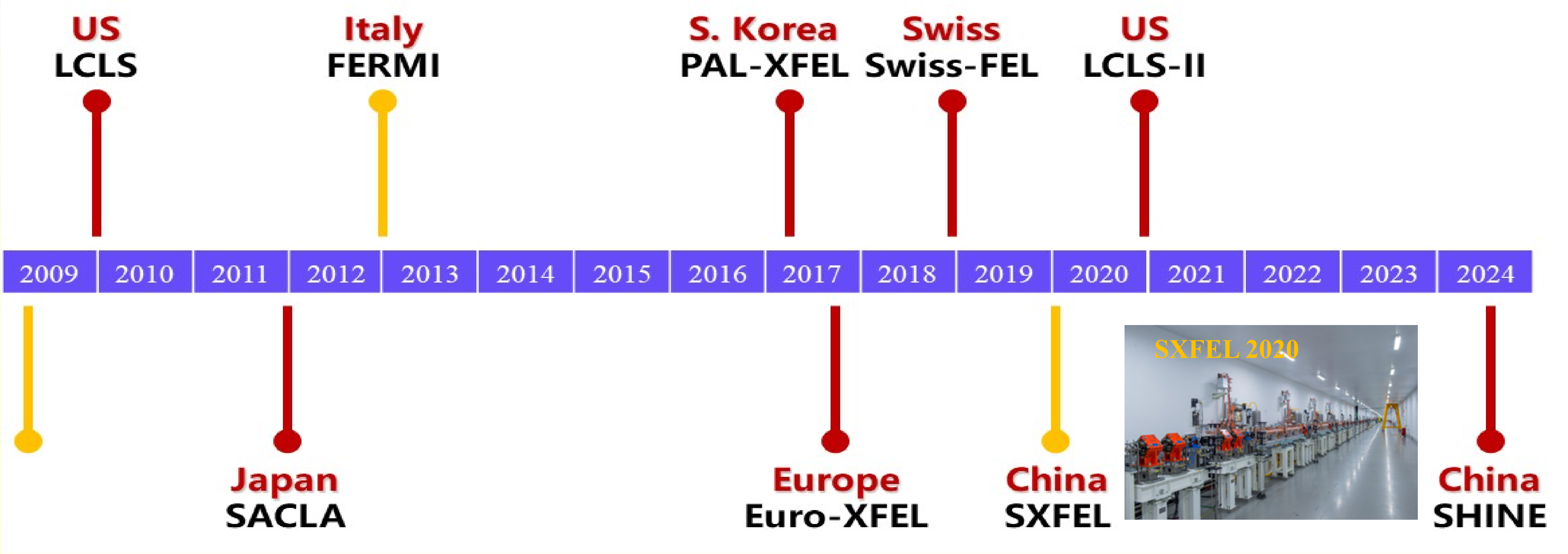
$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

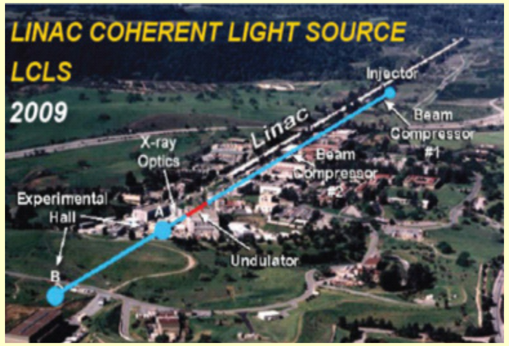
Gain length

$$L_G^{1D} = \frac{\lambda_u}{4\sqrt{3}\pi\rho} \qquad L_G^{3D} = (1+\eta)L_G^{1D}$$

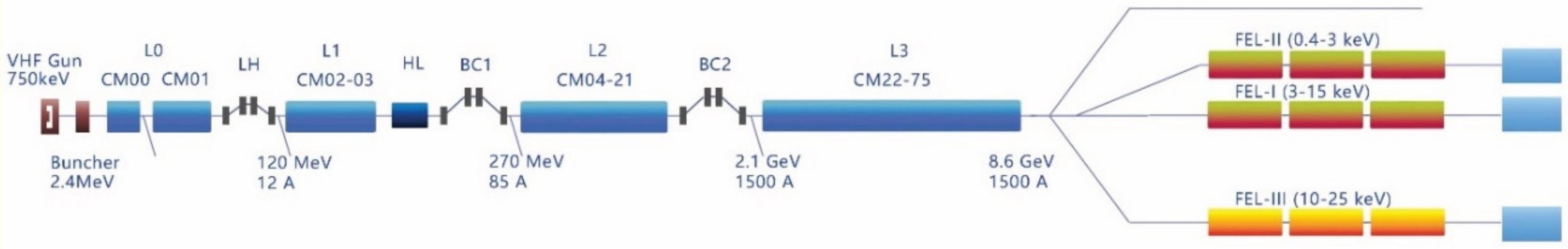
- **Synchrotron radiation** emitted from the stored electrons in a storage ring is continuous spectrum, high average power, high repetition rate, high brightness (10^{23}) and with multi-beam ports.
- **FEL** generates from undulators radiation is advantage in its monochromaticity, coherence, short pulses (~fs), high pulse power and even higher brightness (10^{33}).
- SR and FEL are **complementary in scientific applications**.

5.3 XFELs in the World





Shanghai HIgh repetition rate XFEL and Extreme light facility（SHINE）



Summary

- As tools of discovery, particle accelerators have been rapidly developed since 1930’s.
- The purpose of accelerator physics is to study behaviors of particle beams.
- The accelerator physics and technology are closely related in design, construction and performance of accelerators.
- Vigorously growing of synchrotron radiation facilities and free electron lasers has also promoted the development of accelerator physics and technology.

Bibliography

1. E. Wilson, An Introduction to Particle Accelerators
2. D. Edwards & M. Syphers: An Introduction to the Physics of High Energy Accelerators
3. S.Y. Lee: Accelerator Physics
4. A.W. Chao and M.Tigner, Handbook of Accelerator Physics and Engineering
5. M. Conte & W. MacKay: An Introduction to the Physics of Particle Accelerators
6. M. Livingston & J. Blewett: Particle Accelerators
7. M. Reiser: Theory and Design of Charged Particle Beams

Thank You for Attention